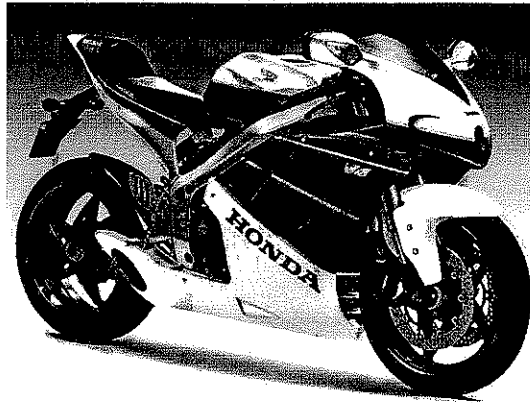


## Motorcycle Dynamics 2013

### Question 1

You are working as part of a design team responsible for the specification of a new 800cc V4 road going superbike. Your role as part of this team is to specify the suspension requirements for the motorcycle. You are given the following information from which to come up with your draft suspension specifications.



Total mass – 158kg  
CofG bias – 60% front  
Front unsprung mass – 9kg  
Rear unsprung mass – 12.4kg  
Rear link ratio – 2.35 (average)  
Rake angle – 22.3 degrees

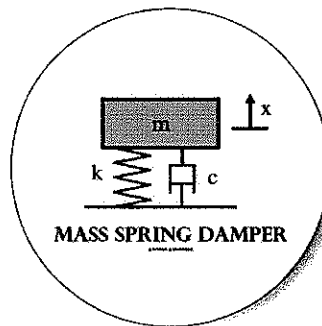
You are expected to make reasonable, justified assumptions for the other parameters you need based on the design requirements of the type of bike being considered. Using this information, provide the actual spring rates, compression and rebound damping requirements for the forks and the shock.

(15 marks)

Assuming that during a track testing session, the front potentiometer measuring the fork movement develops a fault. Show in detail how you would go about trying to re-create a front suspension displacement channel from the changes in wheel speed of the bike.

(5 marks)

## Question 2



The figure above shows a simple mass  $m$  attached to a spring of stiffness  $k$  and a viscous damper with a damping coefficient  $c$ . The displacement,  $x$ , is given by

$$\frac{d^2x}{dt^2} + 2\zeta\omega \frac{dx}{dt} + \omega^2 x = 0$$

where  $\omega = \sqrt{\frac{k}{m}}$  and the viscous damping factor  $\zeta = \frac{c}{2m\omega}$

- Describe what you understand by the term 'Ride Frequency' (3 marks)
- What do you understand by the phrase 'Critical Damping'? (2 marks)
- Describe how the 'Damping Ratio' can be used to define damping characteristics. (3 marks)

- Show that the differential equation shown above, is exactly equivalent to the normal form used in motorcycle dynamics ' $ma + cv + kx = 0$ ', and solve this differential equation for  $\zeta = 1$  where:
- $x = (A + Bt)e^{rt}$

(12 marks)

### Question 3

- a. Considering that wheel velocities are an integral part of any modern control system, what precautions have to be made when utilizing the raw data from these sensors?

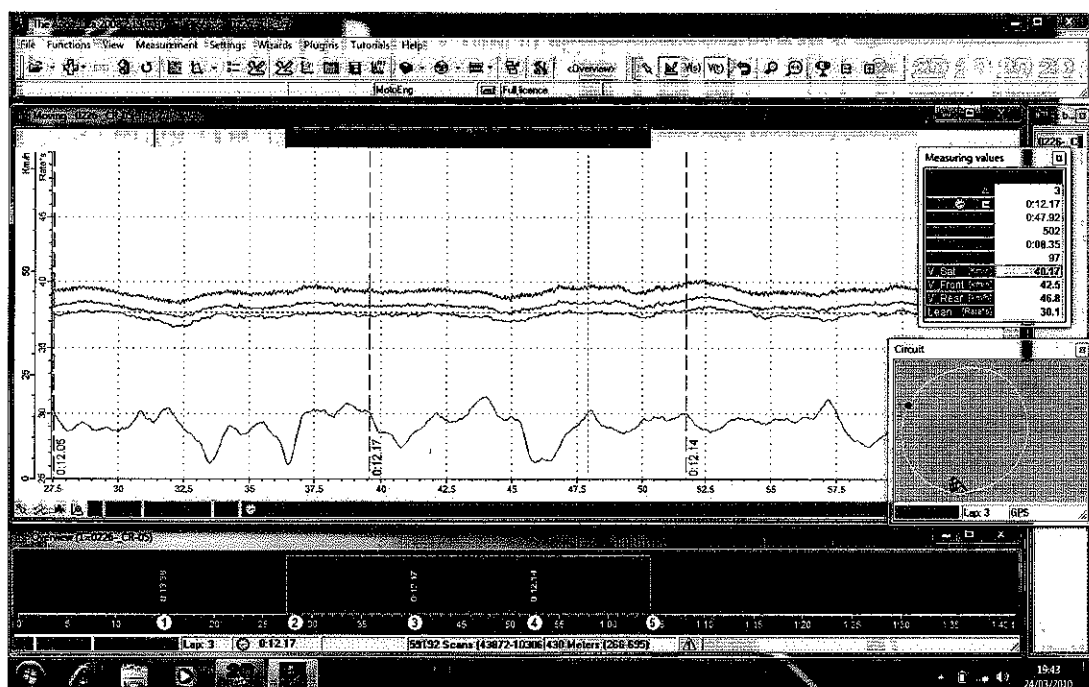
(5 marks)

- b. Go on to explain what sensors might be required and how they would be implemented in modern control system.

(5 marks)

- c. Derive an equation, which expands on the concept of the ideal lean angle to include the motorcycles average tyre thickness into its required lean angle to give a more accurate representation of the required lean angle.

(5 marks)



- d. For the motorcycle show in the above data trace, calculate the ideal lean angle and the 'real' lean angle assuming the test track diameter to be 45m and realistic values for the required motorcycle geometry and compare your solution to the data on the trace and discuss your results.

(5 marks)

#### Question 4

- a. Derive a set of equations that can be used to determine the wheel loads of a motorcycle in both static and dynamic situations.

(8 marks)

Consider a motorcycle with the following properties:

Mass with rider,  $m = 225\text{kg}$

Reduced Mass,  $m^* = 245\text{kg}$

Frontal Area,  $A = 0.75\text{ m}^2$

Drag Coefficient,  $C_D = 0.7$

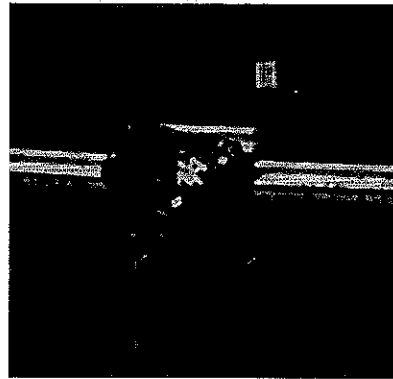
Wheelbase,  $p = 1.4\text{m}$

50/50 weight distribution

CofG height,  $h = 0.65\text{m}$

Front fork stiffness  $k_f = 22\text{ N/mm}$

Rake angle =  $22^\circ$



- b. What is the *total* driving force required to maintain the motorcycle at a constant velocity of  $155\text{ kmh}^{-1}$  along a horizontal flat road?

(3 marks)

- c. If the motorcycle has a maximum power output of  $56\text{KW}$ , what would you expect its aerodynamically limited top speed to be?

(3 marks)

- d. For this motorcycle in braking, its data logger shows a deceleration of  $0.8g$ , with a suspension compression of  $95\text{mm}$ . Using the data provided above, show using calculations that this is a reasonable suspension compression value.

(6 marks)



## EXAMINATION SOLUTIONS : SESSION 2012/13

|              |                             |       |         |
|--------------|-----------------------------|-------|---------|
| Programme:   | BEng Motorcycle Engineering | Year: | Level 6 |
| Module/Unit: | Motorcycle Dynamics         |       |         |
| Author:      | Dr Owen Williams            | Page: | 1 of 7  |

| Question No | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Marks |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| Q1          | <p>Assumptions: Rider mass (<math>\sim 75\text{Kg}</math>), Ride freq <math>\sim 2.1\text{Hz}</math></p> <p>Front Spring mass = <math>(0.6 \times (158 + 75)) - 9 = 130.8\text{Kg}</math></p> <p>Rear Spring mass = <math>(0.4 \times (158 + 75)) - 12.4 = 81.2\text{Kg}</math></p> <p>Using <math>f = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \Rightarrow K = (2\pi f)^2 \times m</math></p> <p><math>K_f = 22772.3\text{ N/m}</math>    <math>K_r = 14136.9\text{ N/m}</math> (vertically)</p> <p><math>C_{crit} = 2\sqrt{Km} \Rightarrow C_{crit f} = 3451.7\text{ Ns/m}</math><br/> <math>C_{crit r} = 2142.8\text{ Ns/m}</math></p> <p><math>\zeta = \frac{c}{C_{crit}}</math> (assume Zeta <math>\sim 0.3</math> to <math>0.5</math>)</p> <p><math>C_f = 1380.68</math>    <math>C_r = 857.1\text{ Ns/m}</math> (Vert)</p> <p><math>K_{fork} = \frac{1}{2} \times 22.77 \times \cos^2(22.3) = 9.75\text{ N/mm}</math></p> <p><math>K_{shock} = 14.13 \times 2.35^2 = 78\text{ N/mm}</math></p> <p><math>C_{fork} = \frac{1}{2} \times 1380.68 \times \cos^2(22.3) = 590.9\text{ Ns/m}</math></p> <p><math>C_{shock} = 857.1 \times 2.35^2 = 4733.3\text{ Ns/m}</math></p> <p>(Both specific for Rebound)</p> |       |

## Continuation Sheet

|              |                     |       |        |
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| Module/Unit: | Motorcycle Dynamics | Page: | 2 of 7 |
|--------------|---------------------|-------|--------|

| Question No | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | Marks                                                                                                                                 |
|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------|
| 1 cont      | <p>Compression Damping (front) <math>C_{air} = 2\sqrt{K_m}</math> (<math>m = unsprung</math>)</p> <p><math>C_{air_f} = 905.4 \text{ Ns/m} \Rightarrow C_f = 362.2 \text{ Ns/m}</math></p> <p><math>C_{fork} = \frac{1}{2} \times 362.2 \times \cos^2(22.3) = 155 \text{ Ns/m}</math></p> <p>Comp Damping (rear) <math>C_{air_r} = 837.4 \text{ Ns/m}</math></p> <p><math>C_{rear} = 334.9 \text{ Ns/m}</math> <math>C_{shock} = 1849.8 \text{ Ns/m}</math></p> <hr/> <p>Use wheel speed, <math>\frac{dv}{dt} \Rightarrow</math> acceleration</p> <p>from <math>F=ma</math>, we can deduce the braking/acc force</p> <p>this gives us <math>N_f</math> and <math>N_r</math> (loads on wheels)</p> <p>from <math>N_f, N_r</math> we use <math>F=Kx</math> to get <math>x</math></p> <p>which is spring displacement</p> <hr/> <p style="text-align: right;">Total</p> | <div style="text-align: right;">15</div> <hr/> <div style="text-align: right;">5</div> <hr/> <div style="text-align: right;">20</div> |

# Continuation Sheet

|              |                     |       |        |
|--------------|---------------------|-------|--------|
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| Question No | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Marks |
|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| 2/a,        | Ride frequency $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ or $\omega = \sqrt{\frac{k}{m}}$ is the undamped natural frequency of the spring body in Ride: higher $f$ = Stiffer                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | (3)   |
| b,          | Critical damping is when the spring in a system is damped such that it returns to its rest position without overshoot in the shortest possible time.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | (2)   |
| c,          | Damping ratio defines system behaviour<br>$\zeta > 1$ = Overdamped<br>$\zeta = 1$ = Critical<br>$\zeta < 1$ = Underdamped.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | (3)   |
| d,          | $\frac{d^2x}{dt^2} + 2\zeta\omega \frac{dx}{dt} + \omega^2 x = 0$ $\omega = \sqrt{\frac{k}{m}} \quad \zeta = \frac{c}{2\sqrt{km}}$ $\frac{d^2x}{dt^2} + \frac{2c\omega}{2\sqrt{km}} \frac{dx}{dt} + \frac{kx}{m} = 0 \Rightarrow \frac{d^2x}{dt^2} + \frac{c\sqrt{k/m}}{\sqrt{km}} \frac{dx}{dt} + \frac{kx}{m} = 0$ $\Rightarrow \frac{d^2x}{dt^2} + \frac{c}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$ <p>To Solve use <math>r^2 + 2\zeta\omega r + \omega^2 = 0</math><br/> quadratic formula <math>\Rightarrow r = -\zeta\omega \pm \omega\sqrt{\zeta^2 - 1}</math><br/> Roots are real in equivalent <math>\zeta = 1</math><br/> Use <math>x = (A + Bt)e^{rt}</math>, sub for <math>r</math><br/> No initial conditions so Solution complete</p> | (6)   |
| Total       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 20    |

## Continuation Sheet

|                     |                            |              |        |
|---------------------|----------------------------|--------------|--------|
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| <b>Question No</b> | <b>Question</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | <b>Marks</b> |
|--------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|
| 3a                 | <p>When using wheel velocities as an input certain factors can have a detrimental effect to how the system operates.</p> <p>Wheel size is important, If different tyre brand or even construction tyres are used they normally won't have the same profile, this will in turn give a false slip percentage due to different the circumference reference being difference, unless the control system has different inputs for different tyres.</p> <p>Care has to be taken with signal pickup, Making sure that the sensor is picking up the correct number of triggers and not getting any abnormal readings, such Allen bolt heads giving double pulses</p> <p>Sensor failure also has to be considered. If the sensor starts to give 'noise' or just stops working altogether the system has to have fail safes and be able to detect irregular input signals</p> <p>The tyre profile has to be considered as well. As the bike leans over the profile ratio between the front tyre and rear tyre will change. Some tyres 'grow' as higher speeds are reached, the centrifugal force of the tyre spinning causing the tyre to deform effectively changing the tyre diameter. (a good example of tyres 'growing' can be seen on Drag cars)</p> <p>Outside factors have to be considered when using wheel speed for and input for a control system. The controller could misinterpret certain dynamic motorcycle behavior<br/>i.e. When wheelie occurs you would expect the front wheel speed to slow, this could be miss interpreted by the system for wheel slip.</p> <p>If the motorcycle goes over 'rough' ground or a curb on a race track you would expect to see the front wheel to slow and rear to slip as the tyres 'roll' over the edges of the kerbs, again giving the system an incorrect view of what the motorcycle is doing</p> | (5)          |

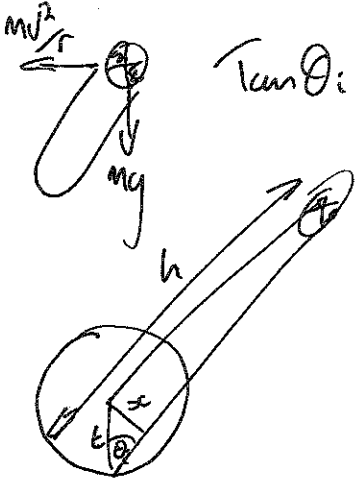
# Continuation Sheet

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| Question No | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | Marks |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| 36          | <p>In an advanced motorcycle control system there are an array of sensors that can be implemented. Gyro, Accelerometers, Front and rear velocity, Torque sensor, Stroke sensors, Throttle position, Butterfly position, RPM, Gear Position. The more sensors used together the more complicated and accurate control systems can be created.</p> <p>Using the Gyro you can determine the lean angle of the motorcycle creating the possibility to determine whether the bike is on its left or right side. This can be useful in racing to where if there were a dominant number of corners I.E 9 right turns to 3 left turns. You would expect the grip levels on the right side to be different to that on the left thus tailoring the control to the track.</p> <p>The accelerometers can be used to calculate the pitch rate of the motorcycle and with a vehicle model input into the ECU you would be able to detect when the front wheel has left the ground and calculate how high the wheel is off the track, after some analysis you would be able to determine where the best acceleration for the bike is and use the pitch rate to help control the bike.</p> <p>Wheel speed will can be used to monitor the slip between the front and rear tyre, part of the control system will be using this data as part of the condition when to reduce the torque delivered to the rear wheel. Another condition that can be used by the ECU can be calculated from the slip % calculation. By deriving the slip you can determine the rate that the slip is increasing and again control the slip appropriately.</p> <p>Torque sensor:- Normally this sensor would give the ECU real time torque delivery allowing a more accurate control over sliding, wheelie and braking.</p> <p>Butterfly position. As most modern motorcycles now have motors controlled by the ECU. It is important to be able to detect the % of opening. Depending on how much torque is required the ECU will determine how much to open the butterflies. Stroke sensors will be used to distinguish the difference between slip and wheelie.</p> <p>Wheel speed can also allow the ECU to determine distance traveled. By using this in conjunction with lap trigger and throttle it should be possible to separate the lap into segments giving greater adjustability. Gear position:- This has a great use as it allows the a ECU to have different Fuel and ignition maps, TCS, anti-wheelie, engine brake. The amount of torque being delivered to the rear wheel will change in each gear due to the reduction ratio thus giving different requirements of % torque reduction for a given situation.</p> | 5     |

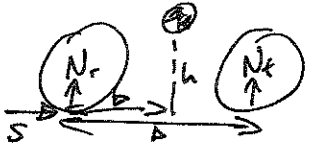
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| Question No | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Marks                 |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------|
| 3c/         |  $\tan \theta_i = \frac{mv^2}{r \cdot mg} \Rightarrow \theta_i = \tan^{-1} \left( \frac{v^2}{rg} \right)$ $\text{or } \theta_i = \tan^{-1} \left( \frac{v^2 r}{g} \right)$ $\sin \theta_i = \frac{x}{t} \quad \sin \Delta Q = \frac{x}{h-t}$ $t \sin \theta_i = (h-t) \sin \Delta Q$ $\left  \frac{\sin \theta_i}{h-t} = \frac{\sin \Delta Q}{t} \right $ <p>d/ Ideal lean angle <math>\theta_i = \tan^{-1} \left( \frac{(40.17 \times \frac{3}{18})^2}{1.5g} \right)</math></p> $\theta_i = 29.42^\circ$ <p>Real <math>\Delta Q = 3.56^\circ \Rightarrow Q = 32.98^\circ</math></p> <p><math>\theta_i</math> is closer to Real lean at <math>30^\circ</math></p> <p>Discuss tyre thickness effects</p> | <p>(5)</p> <p>(5)</p> |
| Total       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 20                    |

# Continuation Sheet

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| Question No | Question                                                                                                                                                                                                                                                                                                                                                                                                        | Marks |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| 4a          |  $N_r + N_f = mg$ <p>Take moments about C of G.</p> $N_f(p-b) + Sh = N_r b \quad \text{so} \quad N_f(p-b) + Sh = mgb - N_f b$ $N_f p - \cancel{N_f b} + Sh = mgb - \cancel{N_f b}$ $N_f = mg \frac{b}{p} - S \frac{h}{p}$ <p>by same method <math>\Rightarrow N_r = mg \left( \frac{p-b}{p} \right) + S \frac{h}{p}</math></p> | (8)   |
| b           | <p>Total = Drag + friction + gravity (for slope)</p> <p><math>155 \text{ km/h} = 43.05 \text{ ms}^{-1}</math></p> <p>force = <math>\frac{1}{2} \rho C_D A V^2 + 0.02 mg = \underline{490 \text{ N}}</math></p>                                                                                                                                                                                                  | (3)   |
| c           | <p>Power = <math>\frac{1}{2} \rho C_D A V^3</math> Solve for V</p> <p><math>V = 57.8 \text{ ms}^{-1}</math> or <math>208 \text{ km/h}</math></p>                                                                                                                                                                                                                                                                | (3)   |
| d           | $N_f = mg \frac{b}{p} + S \frac{h}{p} = mg \left( \frac{1}{2} \right) + 0.8g \times \left( \frac{0.65}{1.4} \right) \times M$ $N_f = 1923.4$ $N_f = Kx \quad x = \frac{N_f}{K} = 87.4 \text{ mm}$ <p>as forks are at <math>22^\circ</math></p> $x_f = \frac{87.4}{\cos 22} = 94.3 \text{ mm}$                                                                                                                   | (6)   |
| Total       |                                                                                                                                                                                                                                                                                                                                                                                                                 | 20    |